Published March 8, 2018

Corn Yield Response to Plant Density and Nitrogen: Spatial Models and Yield Distribution

Rai Schwalbert,* Telmo J.C. Amado, Tiago A. N. Horbe, Lincon O. Stefanello, Y. Assefa, P.V. Vara Prasad, Charles W. Rice, and Ignacio A. Ciampitti*

ABSTRACT

Understanding the relationship of corn (*Zea mays* L.) yield responses to plant density and nitrogen (N) fertilization is critical to production decisions. The main objectives of this study were to (i) evaluate yield responses to plant density and fertilizer N rate at varying yields adjusting models considering a spatial component, (ii) perform a validation for the fitted models with an independent dataset, and (iii) identify key statistical parameters for the yield data distribution governing response models. Analyses were conducted with information from seven fields with 21 studies (one study per yield environment, with three environments per field) conducted from 2009 to 2017 in southern Brazil with geospatial data collected to evaluate yield response to plant density and fertilizer N rates (28911 data points) and one additional database with 12 field studies conducted from 2012 to 2015 in the US Midwest (1773 data points). Databases were divided into training and validation datasets. Field experiments evaluating both plant density and N rate were selected as training dataset. Key research findings were (i) yield–factor response models were dependent on yield environment and within a yield environment those models remained constant regardless the year, country, and hybrid for all evaluated fields, (ii) statistical models considering spatial correlation of the random errors outperformed those considering errors independent and identically distributed and, (iii) yield distribution with comparable 50% interquartile range and mode portrayed similar yield–factor relationship. In summary, fitting spatial yield–density models considering yield data distribution is critical to upscale site-specific models to larger spatial domains.

Core Ideas

- Corn yield response to plant density and N rate were dependent on yield environment.
- Agronomic optimal plant density and N rate were positively correlated to yield level.
- Yield to density within a yield environment was independent on year, country, and hybrid.
- Similarity in yield frequency data distributions lead to similar yield–factor responses.

Published in Agron. J. 110:970–982 (2018)
doi:10.2134/agronj2017.07.0425
Supplemental material available online

Globally, corn (*Zea mays* L.) yield has been increasing in the last decades, a result of a combination of genetic and agronomic management factors (Duvick and Cassman, 1999; Tollenaar and Wu, 1999; Ciampitti and Vyn, 2014), which are difficult to analyze alone due to the high degree of interaction among them (Duvick, 1997; Tollenaar and Lee, 2002). From all management changes, increases in plant density and the use of synthetic fertilizer N have been two of the main factors responsible for a significant portion of the historical corn yield gains (Duvick, 2005). Plant density is the agronomic factor that changed the most in the last few decades (Tollenaar and Lee, 2002; Assefa et al., 2017) and N represents the most required nutrient for corn (Bender et al., 2013; Ciampitti et al., 2013), frequently limiting yield (Dhital and Raun, 2016; Scharf et al., 2005; Shanahan et al., 2008). Modern corn hybrids have been developed with the ability to support crowding stress but increasing the dependency to N (Ciampitti and Vyn, 2012; Tokatlidis and Koutroubas, 2004). However, even in studies involving modern corn hybrids, varied yield responses to plant density (positive, neutral, and negative) and to N rates are commonly reported, mainly associated with complex interactions between the genotype, environment, and management practices (G × E × M) (Assefa et al., 2016; Hörbe et al., 2013; Koch et al., 2004; Roberts et al., 2012).

Previous research has suggested that agronomic optimal plant density (AOPD) for maximizing yield was related to soil depth (Barnhisel et al., 1996), elevation (Shanahan et al., 2004), water supply (van Averbeke and Marais, 1994) and soil type (Woli et al., 2014). Since all above-mentioned studies have reported a positive correlation between corn yield and AOPD, it is adequate to assume that corn yield level may be used as a proxy to assess the optimal plant density, regardless of the yield limiting factors (G × E × M), as summarized by Assefa et al. (2016).

Different from yield–plant density, yield–N rate relationship is not directly dependent on yield (Arnall et al., 2013; Raun

R. Schwalbert, Agricultural Engineering Dep., Federal Univ. of Santa Maria, Rural Science Center, Santa Maria, RS, Brazil; R. Schwalbert, T.J.C. Amado, T.A.N. Horbe, L.O. Stefanello, Soil Dep., Federal Univ. of Santa Maria, Rural Science Center, Santa Maria, RS, Brazil; T.J.C. Amado, Y. Assefa, P.V.V. Prasad, C.W. Rice, I.A. Ciampitti, Dep. of Agronomy, Kansas State Univ., 2004 Throckmorton Plant Science Center, Manhattan, KS 66506. Received 30 July 2017. Accepted 5 Jan. 2018. *Corresponding author (rai.schwalbert@gmail.com, Ciampitti@ksu.edu).

Abbreviations: AONR, agronomic optimum nitrogen rate; AOPD, agronomic optimum plant density; BR, Brazil; EONR, economically optimum nitrogen rate; HYE, high yield environment; IQR, interquartile range; LYE, low yield environment; MYE, medium yield environment; RCB, randomized complete block; RMSE, root mean square error; US, United States.

et al., 2011). The soil N supply, which is highly influenced by seasonal weather conditions and soil characteristics (Ferguson et al., 2002), plays a relevant role influencing yield–N rate response (Raun et al., 2011; Scharf et al., 2006). Following this rationale, a mass approach (yield goal × constant factor-N credit) is inadequate to predict the agronomic optimal N rate (AONR) for corn (Scharf et al., 2006).

A more comprehensive evaluation of corn yield response to plant density and fertilizer N rates and how those models are influenced by varying yield environments is crucial to corn production. Thus, the objectives of this study were to: (i) evaluate yield responses to plant density and fertilizer N rate at varying yields adjusting models considering a spatial component, to compare standard statistical models (assuming errors independent and identically distributed [i.i.d.]) versus models considering spatial correlation of the random errors, and (ii) validating the yield–factors (density and N rate) models utilizing completely independent datasets (validation data), collected from different years, locations, and countries (Brazil and United States). Since our data were obtained from different years (2009–2017), across locations (Brazil and United States) and considering geospatial data, totalizing an amount of 30684 points, a third objective was pursued with the goal to (iii) build a link between yield–density models and the yield data frequency distribution through the analysis of statistical parameters (e.g., mode, skewness, quartiles) to evaluate those as potential indicators of the most probable yield–plant density relationship for a given dataset and to evaluate the potential to upscale these site-specific models to larger spatial domains.

MATERIAL AND METHODS

Seven field experiments (with 3 studies per field, 21 total) conducted from 2009 to 2017 in southern Brazil were used in this study (Fig. 1A). Experiments were conducted following a hierarchical approach. As a first step, fields were classified in yield environments based on the average corn yield of previous yield monitor information (four prior corn seasons from the same field). Weather information from those years are presented in supplemental Fig. S1 (see online version to access supplemental material). The yield maps were interpolated using a 10 × 10 m grid size resolution and overlapped. Yield average of each pixel was calculated. From this step, three yield environments were identified and classified as low (<10 Mg ha^{-1}), medium (10 and 13 Mg ha^{-1}), and high (>13 Mg ha^{-1}). Yield environment classification was performed following the synthesis–analysis for corn yield response to seeding rates performed by Assefa et al. (2016). Seeding rate by fertilizer N studies were established in each yield environment. Thus, each field presented three complete studies considering all yield environments.

Seven research studies were grouped according to the known source of variation. For Group I, two site–years per yield environment (6 total) were represented by studies portraying a strip-design with a factorial arrangement (seeding rate × fertilizer N rate), conducted during two growing seasons, 2014–2015 and 2016–2017, with three replications. The treatment levels for the seeding rate were 56,000; 64,000; 72,000; 80,000; and 88,000 plants ha^{-1} (P1630H, Pioneer hybrid) and the fertilizer N rates were 0, 60, 120, 180, and 240 kg N ha^{-1} (Urea, 46% N). All the plots received 20 kg N ha^{-1} at planting (except for the control). For fertilizer N rates lower than 120 kg N ha^{-1}, all N was applied at the V4 stage; for the rest of the N, 120 kg N ha^{-1} was applied at V3 and the remainder at the V7 stage. For Group II, three site–years per yield environment (9 total), were represented by the single-factor plant density, conducted during three growing seasons: 2009–2010 (P30F53), 2010–2011 (P30F53), and 2016–2017 (P1630H); all studies presented a randomized complete block (RCB) design with four replications. Plant density levels were equivalent to the ones evaluated in the Group I and for all treatments a total fertilizer N was applied at 200 kg ha^{-1} (Urea, 46% N). For Group III, two site–years per yield environment (6 total), were represented by the single-factor fertilizer N, conducted during two growing seasons, 2014–2015 and 2015–2016; all studies were arranged in a RCB design with four replications, using the same fertilizer N rates (same N source and timing of application), and hybrid relative to Group I. For both hybrids used in this study (P1630H and P30F53), the recommended AOPD ranged from 70,000 to 80,000 plants ha^{-1} for the study region, with a thermal time requirement of approximately 830 growing degree-days. The hybrid P1630H possessed herbicide tolerance and resistance to feeding from certain aboveground insects (Cry1F protein from *Bacillus thuringiensis*).

Table 1 presents descriptive information for each field experiment conducted in Brazil. Plot size was 30 × 100 m, presenting 0.5-m row spacing. Plots were uniformly fertilized with all the recommended nutrients for their respective growing region. At harvest, yield was recorded after hybrids achieved physiological maturity, through mechanical harvest using a combine equipped with grain yield sensor. Corn yield was recorded using a data logger and it was adjusted to 155 g kg^{-1} moisture. The smaller experimental unit was the plot, within the yield environment and within a block. In each plot, multiple values of yield were registered and treated as pseudo-replications during the statistical analysis to avoid a degree of freedom overestimation (Crawley, 2014).

The data from the group I constituted the 'training-data' (two site–years evaluating both density and fertilizer N rate, 6 studies total). Data collected from Groups II and III were employed for validation purposes, herein termed as 'validation-data' [five site–years, three for density (9 studies) and two for fertilizer N rate (6 studies), 15 studies total]. Data grouping into training and validation datasets was necessary to test model replicability irrespective of the difference between the two datasets in space, time, and hybrids. Additionally, a dataset comprising 1773 observations related to plant density trials for corn hybrids (Assefa et al., 2016) conducted from 2012 to 2015 in 5 states in the United States was utilized for validation purposes (Fig. 1B). The yield environment classification for the dataset from the United States was done among fields, instead of within a field, since within-field spatial variability was not available in those trials like the ones utilized from Brazil (Groups I, II, and III). The US database was collected from studies performed in small plots, 3.0 m × 5.4 m long (0.76-m row spacing). Thus, data collected from individual experiments were entirely classified in low, medium or high yield environment based on their mean corn yield per site.

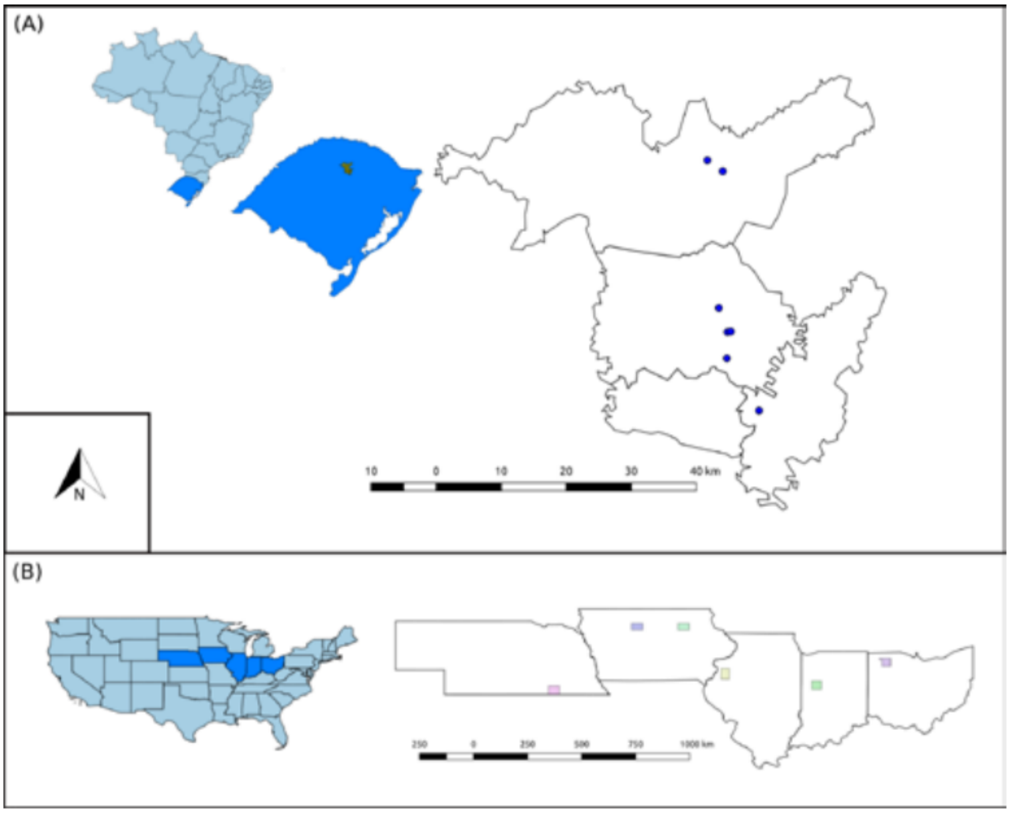

Fig. 1. Spatial geographical distribution for the seven studies performed in Brazil, Rio Grande do Sul state (A), and for the data collected from the United States, coming from five states: Nebraska, Iowa, Illinois, Indiana, and Ohio (B).

Statistical Analyses

Descriptive analyses such as minimum, maximum, standard deviation, mean and mode were conducted using R software (R Core Team, 2017) for both training and validation data. Yield variation accounted by known factors such as fertilizer N rate, plant density, yield environment, location, and interaction among these factors and unknown factors for training data was estimated using the R-package "varComp" (Long, 2015). Hierarchal approaches were followed for data analyses; at first, overall yield response models were built individually for plant density and N rate. In this step, the analyses were run first considering plant density as a fixed effect and the remnant sources of variation as random, and second, considering N rate as a fixed effect and the remnant factors as random. Linear quadratic and quadratic-plateau models were fitted to the least square means (adjusted data) to identify the best model that explains the overall (global) yield–factor relationships.

As a second step, the yield–factor relationship was evaluated by yield environment. For this purpose, the significance of the N rates × yield environment, and plant densities × yield environment interactions were analyzed via ANOVA with a mixed model. Plant density, N rate, yield environment, and their respective interactions were considered as a fixed effect, with block nested into the yield environment factor considered as random effects. The random error was considered potentially correlated under two covariance models: a random block (RB) model, and then a random block model plus spatial (SP) correlation of errors (RB + SP). For the RB + SP. models, exponential, Gaussian, and spherical correlation functions were evaluated using the "nlme" R-package (Pinheiro et al., 2017). These models [RB, RB + SP(Exp), RB + SP(Gau), RB + SP(Sph)] were adjusted with homogeneous and heterogeneous variances for the different yield environments. Model selection for the correlation structure was done following the Akaike information criteria (AIC). When comparing homoscedastic and heteroscedastic models, Likelihood Ratio Test (LRT) was used.

The statistical model took the following form:

$$Y_{ijk} = \mu + N_i + P_j + YE_k + B(YE)_{l(k)} + NxP_{(ij)} + NxYE_{(ik)} + PxYE_{(jk)} + NxPxYE_{(ijk)} + ER_{(il)k} + EC_{(jl)k} + \omega_{ijkl} + \varepsilon_{ijklm} \quad [1]$$

where Y_{ijk} represents the response variable (corn yield)$_{;}$ μ represents the overall mean; N_i represents the N rates ranging from i to n; P_j represents the plant density ranging from j to p; YE_k represents the yield environment ranging from k to q; $B(YE)_{l(k)}$ represents the block effect ranging from l to s nested in the k^{th} yield environment; NxP_{ij}, $NxYE_{ik}$, $PxYE_{jk}$ and $NxPxYE_{ijk}$ represents the fixed effect interactions; $ER_{(il)k}$ represent the row error term; $EC_{(jl)k}$ represents the column error term, ω_{ijkl} represent the random error in the plots; and, ε_{ijklm} represents the random error of the repeated measures. ω_{ijkl} and ε_{ijklm} were potentially considered to be spatially correlated. Location was considered as the block effect since this factor did not improve model fit but, if considered, added a higher degree of parameters and complexity to the overall response model. Similar models were used for Peralta et al. (2015), Peralta et al. (2016), and Córdoba et al. (2016).

When statistical interactions were documented, linear, quadratic and quadratic-plateau models were fitted to the least square means to identify the best models that explain the yield–factor relationship within a yield environment.

Table 1. Characterization of field studies conducted in Brazil related to city/county, average temperature and precipitation during the growing season, organic matter content, clay content, pH, year of study, and soil type for each site–year.†

	Source variation	City/ County	Temp (°C)	Precip. (mm)	OMC (g kg^{-1})	Clay content (g kg^{-1})	pH	Year	Soil taxonomy subgroups
	PD	NMT	28	1254	32	480	5.7	2014–2015	
	PD	NMT	27	1232	32	470	5.9	2016–2017	
Training data	NR	NMT	28	1254	32	510	5.6	2014–2015	
	NR	NMT	27	1232	32	460	5.8	2016–2017	
	PD	NMT	29	1185	34	480	5.8	2009–2010	Rhodic Hapludox
	PD	VG	28	921	31	520	6.0	2010–2011	
	PD	NMT	27	1232	32	470	5.9	2016–2017	
Validation data	NR	CA	29	1054	28	510	6.1	2014–2015	
	NR	CA	27	1350	31	500	6.1	2015–2016	

† PD = plant density, NR = nitrogen rate, OMC = organic matter content, NMT = Não-Me-Toque, VG = Victor Graeff, CA = Carazinho.

The third and fourth steps were the validation analysis; first, the validation data (not used in the models) was superimposed on the overall models and, second, it was superimposed on the specific environment models. Linear and quadratic models were fitted to the validation data to get regression coefficients.

To verify if training and validation datasets were described by the same mathematical model or if two models would be necessary, the following approach was pursued: training and validation datasets were merged in a new data frame and a dummy variable (ID) was added to the data frame to differentiate them (a new column in the data frame). The yield–plant density models were fitted considering corn yield as a dependent variable, plant density as an independent continuous variable and ID as a categorical independent variable. The yield–N rate models were fitted considering corn yield as a dependent variable, N rate as an independent continuous variable and ID as a categorical independent variable. Interaction terms among the coefficients and ID were added, aiming at forcing the model to estimate one coefficient to each dataset (training and validation). Four different models were tested: (i) training and validation data with the same angular coefficient, but different linear coefficients and intercepts (just one interaction term between angular coefficient and ID), (ii) training and validation data with the same linear and angular coefficients but different intercepts, (iii) training and validation data with the same linear, angular coefficients and intercepts (without interaction term), and (iv) training and validation data with different linear, angular coefficients and intercepts (interaction terms between angular coefficient and ID, linear coefficient and ID, and intercept and ID). It allowed us to verify if one or two models would be needed and, if two models were needed, what coefficients would differ between those models. The coefficient comparison is also important because two models could have the same shape (equal linear and angular coefficients) and differ only in the intercept. The models were compared using the AIC.

RESULTS

Overall Corn Yield Responses to Plant Density and Fertilizer N Rates

For the training data (six studies total, two fields), corn yield ranged from 5.1 to 19.8 Mg ha^{-1}, and was not normally distributed ($W = 0.97$, $p < 0.001$), presenting a negative skewness (-0.51) with mean of 12.5 Mg ha^{-1} and mode of 12.6 Mg ha^{-1}. Among the known sources of variation for yield, fertilizer N rate accounted for 9% and plant density for 4%. The proportion of the variance explained by these factors substantially increased when the interactions with yield environment were considered in the analysis. Following this rationale, the analysis by yield environment was pursued with the main goal of improving the understanding of yield responses to the factors evaluated in this study.

Yield variance was explained by two overall models separately considering plant density and N rate as fixed effects. Averaged across all variables (fertilizer N rates, location, yield environments, and blocks), yield response to plant density varied significantly following a quadratic model (Fig. 2). The AOPD, represented by the first derivative equal to zero, was approximately 82,000 plants ha^{-1} (Fig. 2A). Thus, as plant density increased from 56,000 to 82,000 plants ha^{-1}, yield improved from 11.2 to 12.1 Mg ha^{-1}, respectively. Yield response to N fertilization also followed a quadratic-plateau model. The minimum yield was 10.1 Mg ha^{-1} with 0 kg N ha^{-1} and the maximum yield was 12.5 Mg ha^{-1} with 160 kg N ha^{-1}, corresponding to the AONR. When the N rate overpassed 160 kg N ha^{-1}, yield reached a plateau (Fig. 2B).

Despite the significant trends in yield response to plant density and N rates, important variability was observed for yield within each level of those factors (Fig. 2C and 2D). The latter is explained since those models did not take into account possible interactions between the factors and the yield environments, and because the plots were large enough to allow that multiple values of yield (pseudo-replicates) were recorded in each condition, which contributes to increasing the variability for the response variable.

Yield–Factor Responses by Yield Environment

Corn yield response to plant density and N fertilization was evaluated based on three yield environments defined according to four previous years of corn yield data (2002/2003, 2006/2007, 2009/2010 and 2012/2013 seasons) collected from the same field (Fig. 3A). For the low yield environment (LYE), average yield for those past growing seasons was 7.7 Mg ha^{-1} with a mode of 6.8 Mg ha^{-1}; for the medium yield environment (MYE), overall yield was 10.3 Mg ha^{-1} with a mode of 9.1 Mg ha^{-1}; and for the high yield environment (HYE), average yield was 14.2 Mg ha^{-1} with a mode of 12.4 Mg ha^{-1} (Fig. 3B).

Training data was categorized based on the geographical position of each plot within the fields according to the

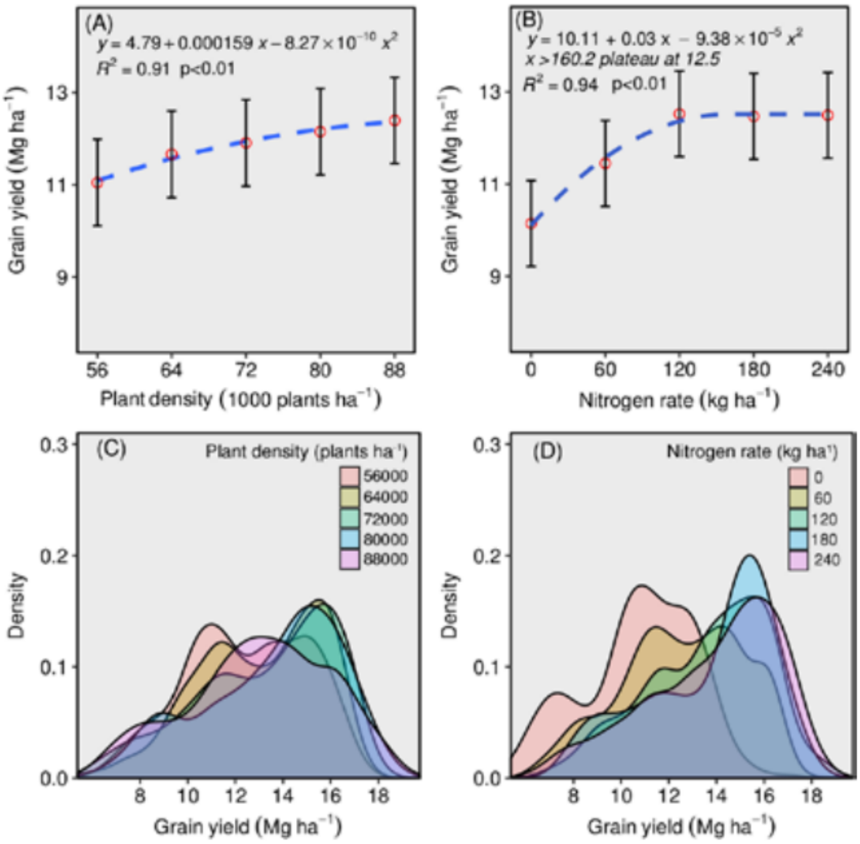

Fig. 2. Overall model for grain yield response to plant density (A) and to nitrogen rates (B). Vertical bar on each data point in the graph is the standard error. Frequency distribution of the plant density levels (C) and nitrogen levels (D) for training data.

previously defined yield environments. For the training data, LYE presented a mean yield of 8.3 Mg ha^{-1} with a mode of 8.5 Mg ha^{-1}; MYE presented an overall yield of 12.5 Mg ha^{-1} with a mode of 11.6 Mg ha^{-1}; and HYE presented a mean yield of 14.5 Mg ha^{-1} with a mode of 15.5 Mg ha^{-1} (Fig. 3C). For the training dataset, the number of observations allocated to each evaluated factor for each yield environment is provided in Table 2.

Significant yield environment by plant density and yield environment by fertilizer N rate interactions were documented (Table 3). Yield environment alone explained 65% of the variation in corn yield. The 35% remnant of corn yield variation was accounted for N fertilization and fertilizer N by yield environment interaction (12%), plant density, and plant density by yield environment interaction (8%), and unknown factors (15%). Furthermore, the random errors were spatially correlated through an exponential structure, and the variance was considered heterogeneous across the yield environments. The model selection was based on (i) the smallest AIC and (ii) the LRT, which indicated that the heteroscedastic model was significantly different from the model considering homogeneous variance among yield environments (Table 3).

Since each factor presented a consistent interaction with yield environment, the yield–factors relationship was dissected by yield environments. For the yield–density relationship, the LYE followed a negative linear trend with yield decreasing at a rate of 23 g plant^{-1} (slope) with each unit of plant density increasing from 56,000 to 88,000 plants ha^{-1} (Fig. 4A). The maximum yield achieved was 8.6 Mg ha^{-1} with a minimum of 7.9 Mg ha^{-1}. For the MYE, yield-to-density relationship followed a quadratic model with a relatively rapid growth region up to 76,000 plants ha^{-1} followed by a slow growth region. In this environment, the maximum yield of approximately 13.3 Mg ha^{-1} was attained at about 88,000 plants ha^{-1} (Fig. 4B). For the HYE, yield-to-density presented a linear positive model with a yield ranging from 13.6 to 15.3 Mg ha^{-1}; and plant density ranging from 56,000 to 88,000 plants ha^{-1}. The HYE followed a positive linear trend with yield increasing at a rate of 54 g plant^{-1} (slope) with each unit of plant density increasing from 56,000 to 88,000 plants ha^{-1} (Fig. 4C).

For the yield–N rate relationship, all yield environments followed a quadratic-plateau trend (Fig. 4D-F). Yield in the LYE increased from 6.8 to 8.7 Mg ha^{-1} when N rate increased from 0 to 131 kg N ha^{-1}, plateauing afterward. In the MYE cluster, as the fertilizer N rate increased from 0 to 165 kg N ha^{-1}, yield improved from 11.1 to 13.6 Mg ha^{-1} followed by a plateau. Lastly, for the HYE, yield improved from 12.2 to 15.5 Mg ha^{-1} as the fertilizer N rate increased from 0 to 177 kg N ha^{-1} also followed by a plateau at higher (>177 kg N ha^{-1}) fertilizer N rates.

Overall Model Validation

The validation datasets (five site–years, 15 studies total) were superimposed on the overall yield–factor (training data) (Fig. 5A and 5B). For plant density validation, two different datasets were utilized (Brazil and United States). In the validation dataset from the US, corn yield ranged from 1.4 to 17.9 Mg ha^{-1}, negative skewed (-0.69) with a mean of 11.8 Mg ha^{-1} and a mode of 13.3 Mg ha^{-1}. In the validation data from Brazil, corn yield ranged from 6.7 to 17 Mg ha^{-1}, presenting a negative skewness (-0.36) with a mean of 12.2 Mg ha^{-1} and a mode of

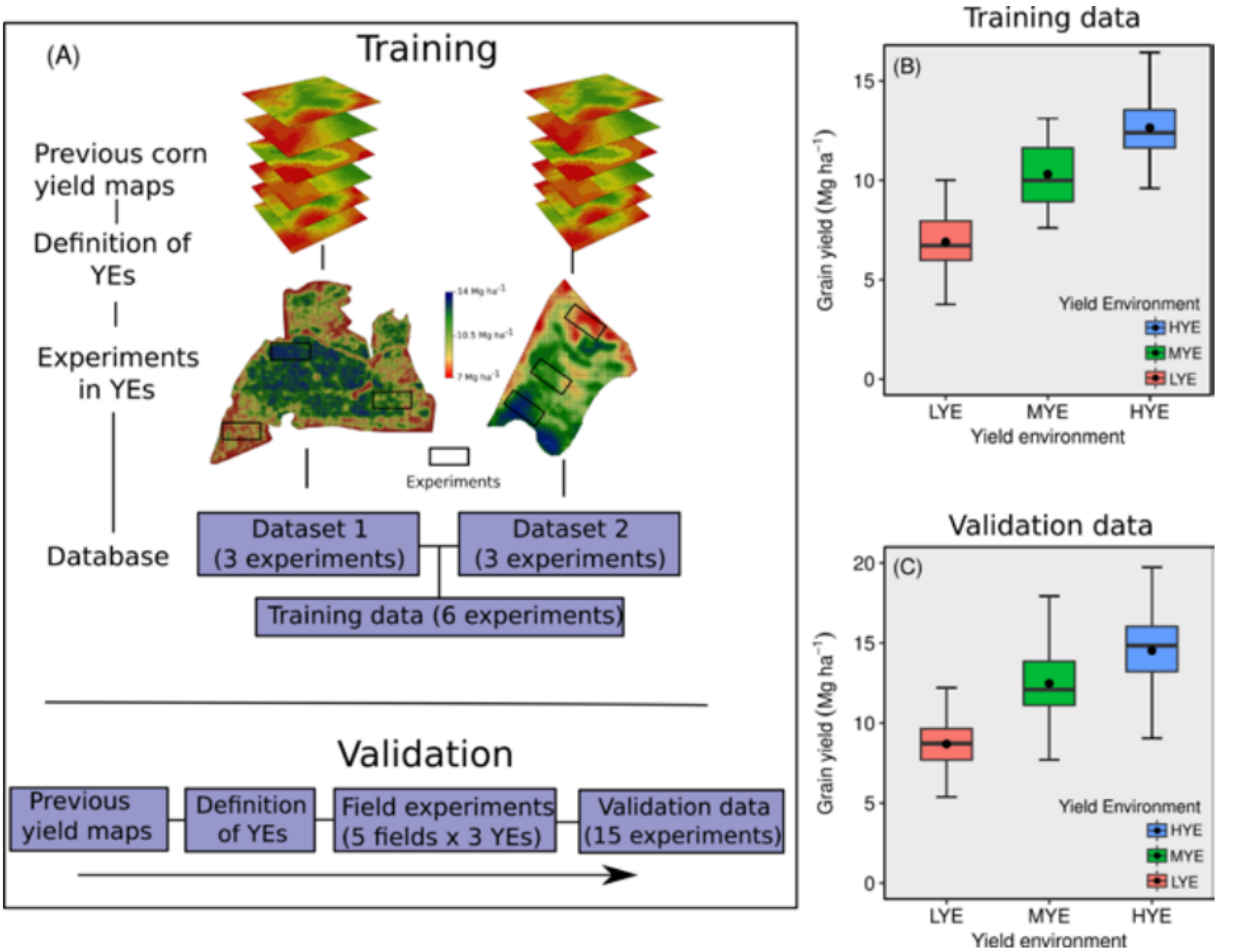

Fig. 3. Framework of yield environment classification based on past-season corn yield data and experiment locations (A). Boxplot of yield derived from training data experiments (B), and boxplot of yield data derived from validation data experiments (C). For boxplot (B, C), the lower and upper hinges correspond to the first and third quartiles (the 25th and 75th percentiles). The upper whisker extends from the hinge to the largest value no further than 1.5 x IQR (inter-quartile range) from the hinge. HYE = high yield environment, MYE = medium yield environment, LYE = low yield environment, and YEs = yield environments.

13.3 Mg ha^{-1} (Fig. 5C). For yield–N rate model validation, only the dataset from Brazil was utilized (2 site–years, 6 studies total). In this dataset, yield ranged from 4.9 to 16.2 Mg ha^{-1}, presenting a negative skewness (-0.54) with a mean of 12.0 Mg ha^{-1}, a median of 12.6 Mg ha^{-1} and a mode of 14.0 Mg ha^{-1} (Fig. 5D). None of the datasets presented a normal distribution ($p < 0.01$).

The root mean square error (RMSE) of the validation data from United States and Brazil for the yield–density model was 2.5 Mg ha^{-1} and 2.7 Mg ha^{-1}, respectively. For the N validation data, the RMSE was 2.6 Mg ha^{-1} relative to the predicted mean. These RMSEs are within the range of the standard deviation of the yield factor that was close to 2.9 Mg ha^{-1}. The overall yield–density relationship did not change among the three different datasets. All regression models presented statistically the same intercept, linear and angular coefficients (Table 4). Similar output for regression models was documented for the overall yield–N rate relationship, without difference among the coefficients according to the AIC.

Site-Specific Model Validation

Validation data were classified by yield environment following the same criteria implemented to the training data (Fig. 3A). All data were superimposed on the site-specific yield–factor models (Fig. 6). For the yield–density relationship in the LYE, the validation data from Brazil followed a similar trend despite differences in the yield levels (slightly superior yields at lower plant densities) (Fig. 6A). Otherwise, MYE and HYE models presented high predictability power (Fig. 6B, C) without presenting differences between regression coefficients between the training and validation data from Brazil (Table 4). High goodness of fit was also documented when the United States validation data were superimposed on the site-specific yield–density models, with no differences in the regressions coefficients at any yield environment. Furthermore, for the yield–N rate relationship the validation data followed the same trend as the model (training data) in the MYE (Fig. 6E), and only presented a difference in the intercept for the LYE and HYE (Fig. 6D and F).

DISCUSSION

Advances in corn breeding have improved the ability of plants to use resources more efficiently and better tolerate crowding stress (Duvick and Cassman, 1999; Tollenaar and Wu, 1999). Consequently, modern corn hybrids have become more responsive to variations in plant density and N rates (Ciampitti and

Table 2. Number of data points in datasets used as training data and validation data. Low yield considered < 10 Mg ha^{-1}, medium yield considered from 10 to 13 Mg ha^{-1}, and high yield considered > 13 Mg ha^{-1}.

Training data				
		Number of data points into each yield environment		
Factor	Level	Low yield	Medium yield	High yield
	0	653	732	635
	60	733	886	1,180
	120	722	1073	1,192
	180	713	913	1,215
N rate (kg ha^{-1})	240	725	1,049	1,258
	56,000	653	920	850
	64,000	675	910	1,163
	72,000	727	829	1,178
Plant density	80,000	712	985	1,147
(plants ha^{-1})	88,000	735	930	1,119
Validation data				
		Number of data points into each yield environment		
Factor	Level	Low yield	Medium yield	High yield
	0	359	345	450
	60	358	391	494
	120	352	402	484
	180	358	407	486
N rate (kg ha^{-1})	240	349	396	485
	56,000	547	583	672
	64,000	568	581	681
	72,000	561	584	685
Plant density	80,000	548	591	669
(plants ha^{-1})	88,000	552	602	692

Vyn, 2012). A positive yield response to plant density and fertilizer N rate was documented in this study, with an optimal value maximizing productivity. Corn yield response to plant density is expected to follow a quadratic model (Assefa et al., 2016; Sangoi et al., 2002; Stanger and Lauer, 2006; Van Roekel and Coulter, 2011), with four major regions of slopes (Assefa et al., 2016) i.e., (i) relatively rapid increase, (ii) slow increase, (iii) no (zero) increase, and (iv) decrease, as plant density progresses from low to high values. Higher plant density leads to increases in intra-specific competition for resources and decreases on individual plant yield (Li et al., 2015; Maddonni and Otegui, 2004; Sangoi et al., 2002), influenced by decreases in both kernel number and weight (Li et al., 2015; Sangoi et al., 2002). Yield response to plant density is determined by a balance between the reductions in per-plant yield and the gains in per-unit-area yield due to the additional plants. The AOPD is achieved when a perfect tradeoff between the per-plant yield reduction and canopy–scale yield gain is reached.

Corn yield response to fertilizer N rates usually presents a positive trend when the soil N supply is inadequate for maximum yield, primarily increasing yield via impact on kernel number and weight (Ciampitti and Vyn, 2011; Moser et al., 2006; Rossini et al., 2012). However, different from the yield–plant density relationship, yield–N rate relationship is not expected the occurrence of a decreasing region immediately after plateauing, but remaining stable as N rate increases. Increments in N rate after this point increases in per-plant N uptake (and

Table 3. Model selection based on Akaike's Information Criterion (AIC) (the lower the better). Likelihood Ratio test was used to check if heteroscedastic component added to model was significant. Bottom section of the table is the analysis of variance of the effect of nitrogen rate, plant density and yield environment for the best model selected (heteroscedastic model considering errors spatially correlated through an exponential structure).

Model selection		
Models†	AIC	*p*-value for Likelihood Ratio Test
RB	11,361.60	–
RB + SP(Exp)‡	10,657.83	–
RB + SP(Gaus)	10,680.56	–
RB + SP(Sph)	10,884.96	–
RB_H	11,332.05	< 0.001
RB + SP(Exp)_H	10,574.20	< 0.001
RB + SP(Gaus)_H	10,593.22	< 0.001
RB + SP(Sph)_H	10,787.77	< 0.001
Analysis of variance		
Source of variation		Prob. > F value
Fertilizer N rate		< 0.001
Plant density		< 0.001
Yield environment		< 0.001
Fertilizer N rate × Yield environment		0.002
Plant density × Yield environment		0.001
Fertilizer N rate × Plant density		0.935
Fertilizer N rate × Plant density × Yield environment		0.923

† RB = random blocks; H = heteroscedastic.

‡ RB + SP: random block model plus spatial correlation of errors; Exp = exponential–; Gau = Gaussian; Sph = spherical

plausible in plant N concentration), but with low or no improvement in per-plant yield (Ciampitti and Vyn, 2012). Thus, a quadratic-plateau model has been documented as the best fit to the yield–N rate relationship (Cerrato and Blackmer, 1990; Roberts et al., 2012; Scharf et al., 2005). Decreases in yield could be expected for extreme high fertilizer N rates (e.g., more than 280 kg N ha^{-1} with yields below 10 Mg ha^{-1}) (Eck, 1984).

In this study, both plant density and N rates positively impacted yield, but no interaction was documented. The latter outcomes are in agreement with several others studying plant density and N rates in corn (Al-Kaisi and Yin, 2003; Blumenthal et al., 2003; Bruns and Abbas, 2005; Ciampitti and Vyn, 2011; Ping et al., 2008; Shapiro and Wortmann, 2006). Since plant N uptake is known to be correlated with corn yield (Ciampitti and Vyn, 2012; Setiyono et al., 2011), and increases in plant density are normally associated with increases in yield (Sangoi et al., 2002; Tokatlidis and Koutroubas, 2004), the logical explanation for the nonsignificant plant density by N rate interaction relies on an increase in N recovery efficiency (kg increases in N uptake per kg N applied). Ciampitti and Vyn (2012) in a review of 100 studies involving plant density and N rate documented low variation in the plant N uptake (g N plant^{-1}) as plant density increases; therefore, per-unit-area N uptake (kg ha^{-1}) rises as plant density increases. When plant density resulted in increases in corn yield, increases in N recovery efficiency have also been reported (Shapiro and Wortmann, 2006; Yan et al., 2017).

Fertilizer N rates (0–240 kg ha^{-1}) more effectively improved yields as plant density increased (56,000–88,000 plants ha^{-1}).

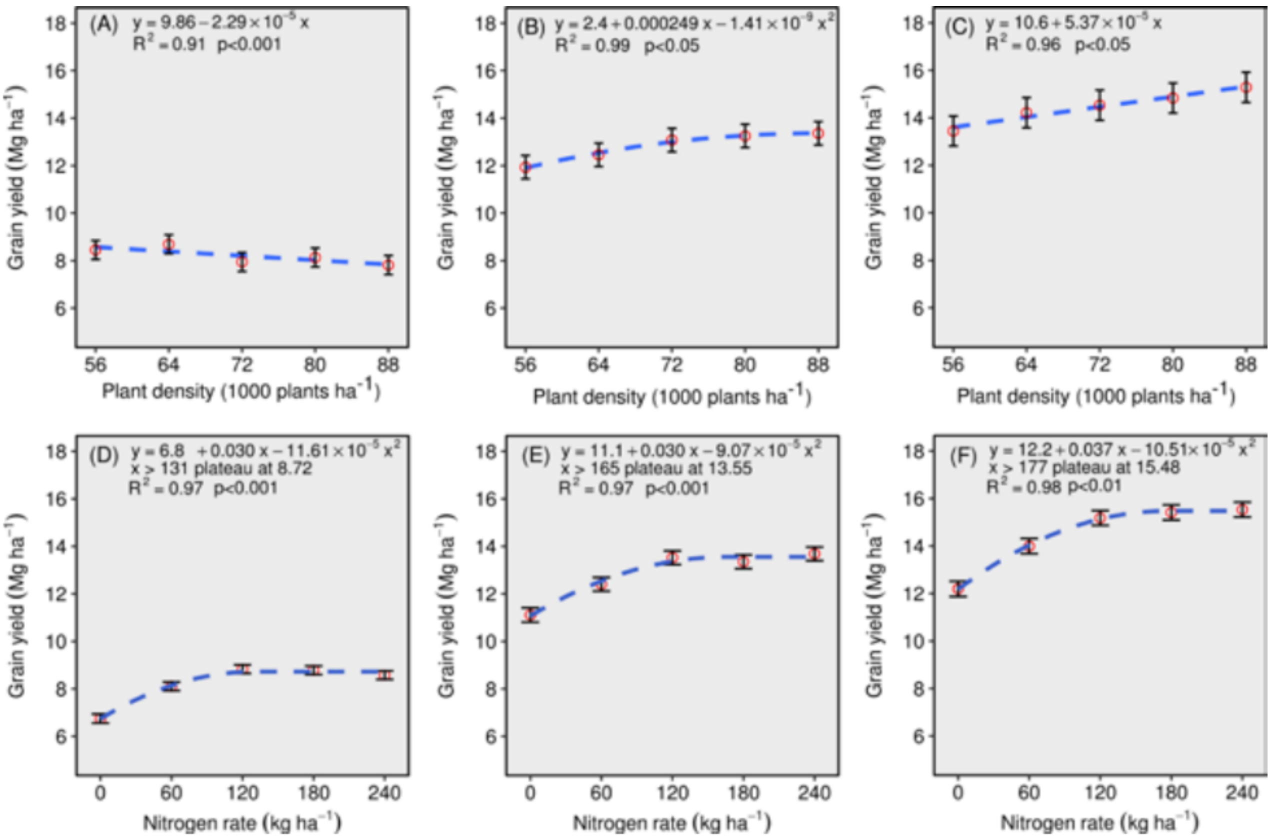

Fig. 4. Models for grain yield response to plant density and fertilizer N rates in three yield environments, i.e., (A and D) low yield environment, (B and E), medium yield environment, and (C and F) high yield environment. Vertical bar on each data point refers to the standard error.

This is in accordance with other studies involving the same production factors in corn (Ciampitti and Vyn, 2011; Shapiro and Wortmann, 2006). Two main reasons can explain the pattern above: (i) considering the lowest N rate and plant density level, it is more likely that soil N supply limited corn yields; and (ii) the strong interaction between plant density and yield environment was documented (Table 2). For plant density, yield–density relationship presented a change in the response trend from negative to positive, from low to high yielding environments. However, N rate did not present the pattern visualized in the yield–density model, but only a change in the model fit (shape) and consequently AONR was documented. The change in response trend possibly acted as a buffer in the overall yield–density relationship because of the antagonistic corn yield response in HYE and LYE. Increases in corn yield in HY were balanced to decreases in LYE conducting to a less step corn yield response to plant density in the overall model.

Corn yield response to plant density and N rate is highly influenced by the environment (Blumenthal et al., 2003; Colville et al., 1964; Inman et al., 2005; Licht et al., 2016). For a yield–density relationship, in the LYE, yield declined with increases in plant density. In this environment, resources such as water, radiation, or nutrients primarily limit yield–density relationship. In the MYE, a quadratic model was the best fit since the limiting resources become less limited allowing a shift of the AOPD to higher plant densities relative to the LYE. Finally, in the HYE there was a linear positive response, the limiting factor to yield could be related to its genetic potential rather than to environmental factors (Assefa et al., 2016).

The primary outcomes on the yield–density are in accordance with previous studies (Assefa et al., 2016; Bullock et al., 1998; Hörbe et al., 2013; Shanahan et al., 2004).

Corn yield response to N rate was also highly influenced by the environment (Inman et al., 2005; Jaynes et al., 2011; Koch et al., 2004; Roberts et al., 2012; Schmidt et al., 2002), but this relationship is not as dependent on yield levels as the yield-plant density model. Corn yield and N fertilizer responses have been documented to be independent (Arnall et al., 2013; Dhital and Raun, 2016; Raun et al., 2011), and both are known to impact N demand. Crop N response is strongly dependent on soil N supply and must be considered to determine the AONR (Dhital and Raun, 2016). Researches have demonstrated the need to adjust a specific N rate by year and location, demanding in-season recommendations (Dhital and Raun, 2016; Franzen et al., 2016; Raun et al., 2005; Schepers et al., 2004; Shanahan et al., 2008). Thus, reactive approaches, including use of remote sensing, should be implemented in addition to predictive approaches, including soil and yield maps, terrain attributes, to predict yield and N response and to determine the economically optimal N rate (EONR) (Roberts et al., 2012; Shanahan et al., 2008). In this study, the yield–N rate model was very similar within a yield environment across independent datasets. This was probably due to the similarities in soil characteristics and weather, since all studies were conducted in the same region in Brazil (Table 1). Therefore, yield–N rate models are restricted to similar soil–weather conditions, crop rotations (e.g., corn–soybean vs. continuous corn), cropping systems

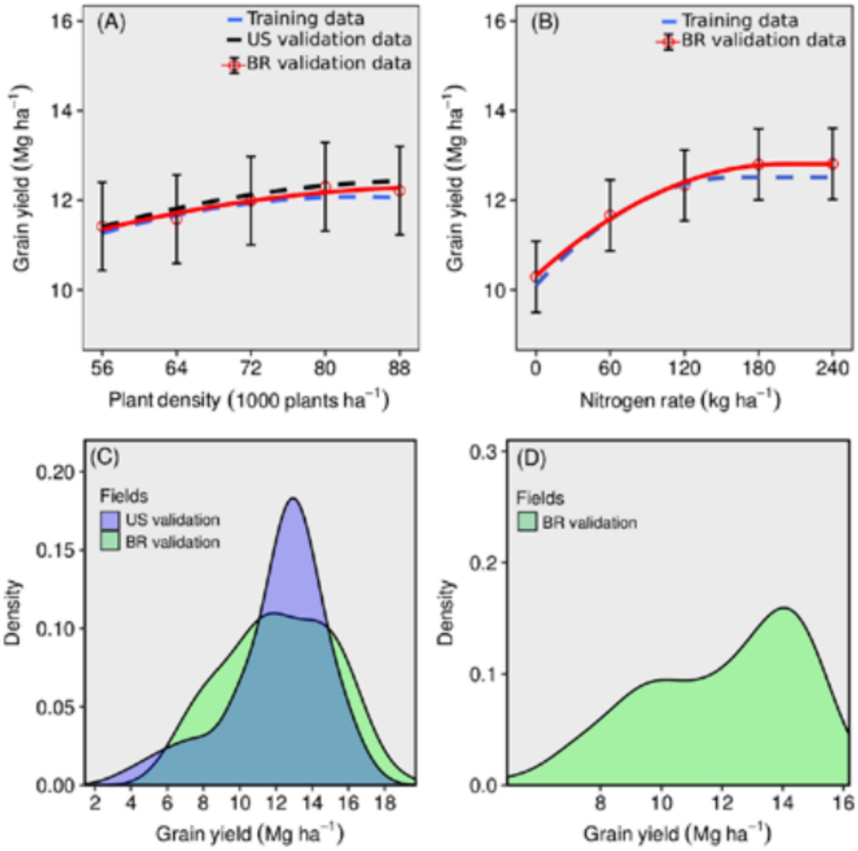

Fig. 5. Overall models superimposed on validation dataset for yield–plant density, two datasets from United States and Brazil (A) and for yield–fertilizer N rates, one dataset from Brazil (B). Vertical bars for each data point refer to the standard error. Frequency distribution of validation data for plant density (C) and for N rates (D). US = United States; BR = Brazil.

(with or without cover crops), and highly dependent on the soil N supply and yield potential per environment.

The choice of yield environment levels to characterize corn yield–density and yield–N rate models can be further improved if yield environments are treated as a continuous variable. From a physiological standpoint, yield–density models should be understood as a continuous change in the response trends, from low to high yielding environments. Following this rationale, all databases were merged to further investigate the yield–density model by dividing yield environments into 2 Mg ha^{-1} yield intervals (Fig. 7A). Thus, for the yield–density relationship theoretically is expected a shift of the AOPD from low to high plant densities as yield increases (Fig. 7 A, B). Yield–density model remained constant within the same yield environment across all databases evaluated (different years, sites, hybrids, and countries; Brazil and United States) when using a plant density range from 56,000 to 88,000 plants ha^{-1}. Hybrid can significantly influence the yield–density model (Assefa et al., 2016; Sarlangue et al., 2007; Widdicombe and Thelen, 2002). One example is that plant density is usually higher for short- rather than for full-season hybrids, because the first ones have small leaf area per plant and leaf area plasticity (Otegui and Melón, 1997), needing more plants to reach the same amount of cumulative intercepted radiation (Edwards et al., 2005). Other example are the studies portraying hybrids less respondent to density benefiting in drought-prone environments, density-neutral corn hybrids (Tokatlidis et al., 2011; Tokatlidis and Koutroubas, 2004).

Yield data distribution for the training and validation databases (Brazil and United States) presented statistically ($p < 0.05$) equal mode, first and third quartiles (Fig. 7C) even when the distributions were not normal. Non-normal corn yield distribution was previously reported by Harri et al. (2009) and Hennessy (2009) in a county level. For data distribution, the aforementioned three factors were determinant for obtaining comparable yield–density models across databases.

This study was not focused on the understanding of the main factors governing the classification of yield environments, but exploring the yield data distribution from different years (2009–2017) and countries (Brazil and United States) to identify statistical parameters in the data highly influencing yield–density models. Therefore, independent datasets could portray in a high-probability similar yield–density models if the following criteria are fulfilled: (i) 50% interquartile range (50%IQR) and (ii) modes statistically similar for the yield data distribution, (iii) plant density evaluated within the same range among datasets, (iv) corn hybrids evaluated are "density-dependent" (Tokatlidis et al., 2011), and (v) modern corn materials, within similar hybrid release years — older corn hybrids present different response to plant density (Ciampitti and Vyn, 2012). Lastly, the yield–factor empirical models presented in this study have limited predictability power as constrained by the tested factors (e.g., plant density range, hybrids, soil characteristics, and yield potential). As previously stated, yield–density models are primarily restricted by the nature of its data distribution and the aforementioned factors such as range of plant density evaluated, yield potential, and genotype characteristics in response to the plant

Table 4. Comparison between training and validation quadratic ($y = ax^2 + bx + c$) models for plant density and fertilizer N rates using Akaike Information Criterion (AIC) as selection criterion (the smaller the value the better the model).

Overall model†			
Coefficient	BR plant density	US plant density	N rate
$a_t = a_v$	11310.1	14172.1	7900.4
$a_t = a_v; b_t = b_v$	11308.3	14171.2	7898.6
$a_t = a_v; b_t = b_v; c_t = c_v$	11306.2	14169.2	7896.3
$a_t \neq a_v; b_t \neq b_v; c_t \neq c_v$	11312.5	14175.6	7902.2

Yield environment model									
	BR plant density			US plant density			N rate		
Coefficient	HYE	MYE	LYE	HYE	MYE	LYE	HYE	MYE	LYE
$a_t = a_v$	3459.4	5214.2	–	3663.9	4775.7	–	2004.1	4114.2	1572.2
$a_t = a_v; b_t = b_v$	3458.1	5213.3	1657.2	3664.8	4754.4	4555.8	2001.2	4112.2	1570.2
$a_t = a_v; b_t = b_v; c_t = c_v$	3456.1	5211.8	1663.2	3662.8	4752.5	4554.1	2006.3	4110.3	1578.5
$a_t \neq a_v; b_t \neq b_v; c_t \neq c_v$	3460.4	5216.5	1660.3	3664.7	4757.4	4556.7	2006.1	4116.1	1573.8

† BR = Brazil; US = United States; HYE = high yield environment; MYE = medium yield environment; LYE = low yield environment (a coefficient was 0 in the plant density regressions in the LYE because the best fit was linear).

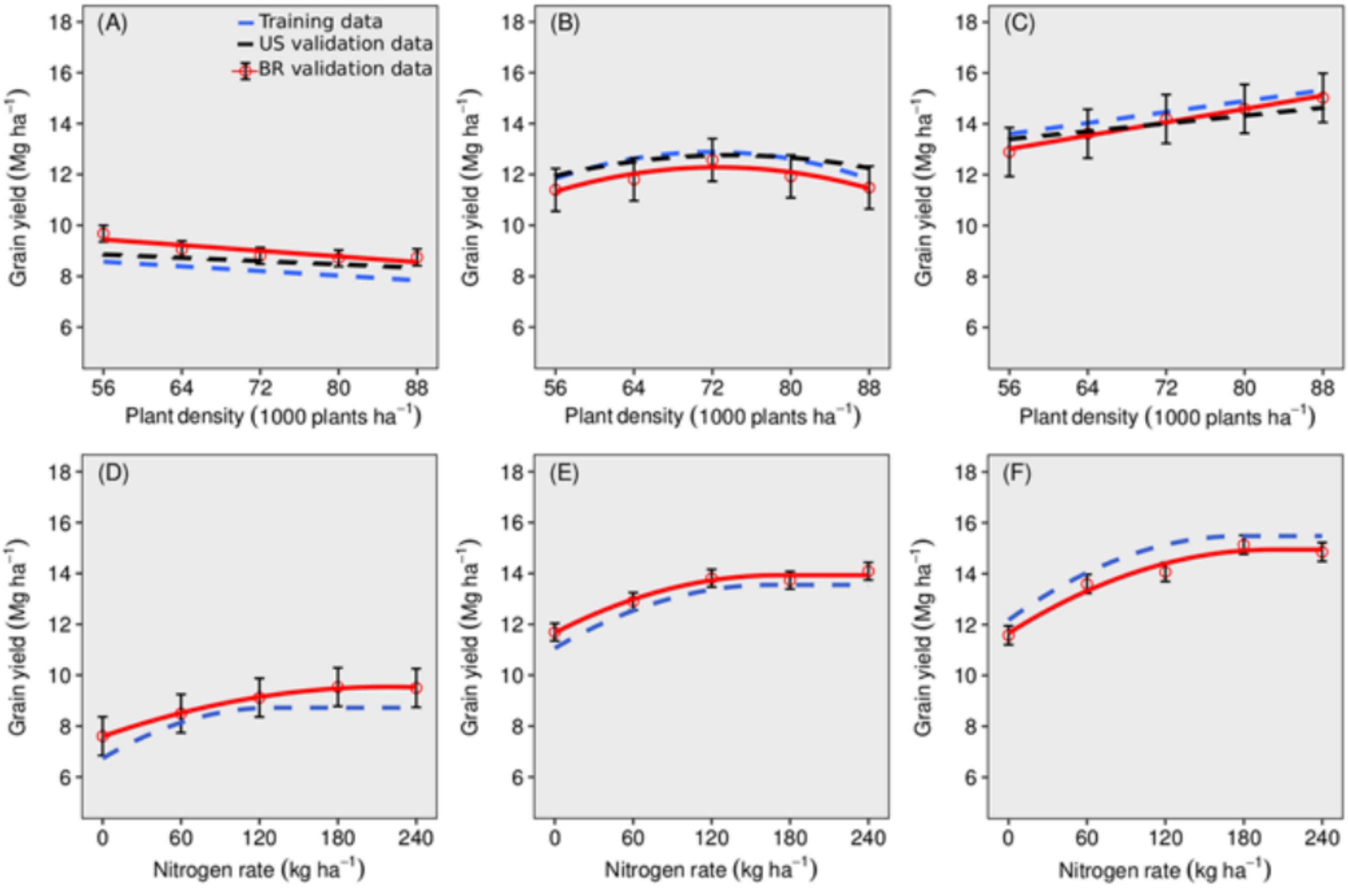

Fig. 6. Quadratic model superimposed on validation dataset by yield environment. i.e., (A and D) low yield environment, (B and E) medium yield environment, and (C and F) high yield environment. Vertical bar for each data point refers to the standard error.

density factor. Development of a more universal yield–density models will provide guidance to improve understanding of potential corn yield gains under both sub- and supra-optimal plant density levels, low- and high-yielding environments, respectively.

In this study previous corn yield maps, from recent past years, were good predictors to in-season yield environments. The yield map data approach to delineate yield environment is considered to be the primary form of precision agriculture technology in the United States (Pierce and Nowak, 1999). However, even at the present times, acquisition of high-quality yield maps is known to be a tedious and educational-intensive task. In this sense, future researches must focus on the development of models combining different data layers to predict in-season yield environments. Nowadays, precision agriculture techniques such as variable seeding (Bullock et al., 1998; Hörbe et al., 2013; Ping et al., 2008) and N rates (Franzen et al., 2016; Holland and Schepers, 2010; Solie et al., 2012) have become more common, in an attempt to account for the within-field variability. Thus, adoption of more universal models to predict corn yield response to factors such as plant density and N rates, can help farmers to improve the efficiency and profitability of

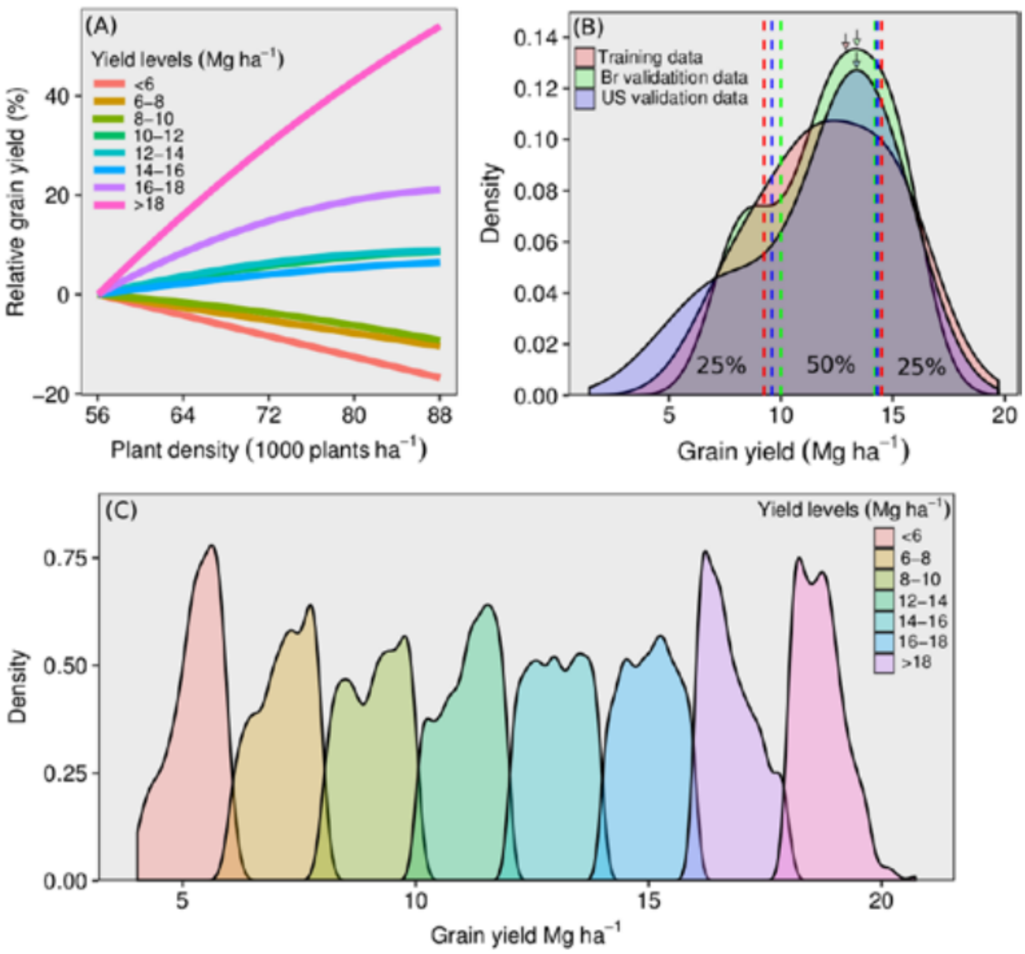

Fig. 7. Yield response by yield environment relative the lowest plant density yield in each environment. Corn yield in the lowest population tested in each environment from <6 Mg ha^{-1}, to >18 Mg ha^{-1} are respectively: 5.4, 7.2, 9.0, 11.1 13.0, 15.0, 16.6, 18.3 Mg ha^{-1} (A). Yield distribution for training and validation datasets, dashed lines represent first and third quartiles, and arrows represent the mode. No statistical difference was documented to mode, first and third quartiles ($p > 0.05$) (B). Yield frequency distribution by yield environment (C).

the decision-making process and the corn production scheme as an integral part of their agricultural activity.

CONCLUSIONS

Yield–density and yield–N rate relationships were largely affected by their interactions with yield environment. For the yield–density relationship, three different trends were documented: a positive linear in HYE, a quadratic in MYE and a negative linear in LYE. For the yield–N rate relationship, comparable fit (quadratic-plateau models) but changing the AONR across all yield environments evaluated. Both AOPD and AONR were positively correlated with yield levels. Yield–factor relationships within the same yield environment remained constant regardless of the year, location (Brazil and United States for plant density) or hybrid evaluated. Since in this study only two hybrids were evaluated, it is not possible to determine if this effect could be extended to different genotypes. Nonetheless, the validation step performed with United States data involved the evaluation of several commercially available hybrids. Since soil N supply is an important factor to be considered for corn yield response to N, more "universal" yield–N rate relationships are primarily more constrained by the study of this factor as compared to the yield–density response models.

In summary, the likelihood of two independent datasets of portraying comparable yield–density response models increase as their yield data distribution becomes more alike, statistically related to position of the mode, and first and third quartiles (50% IQR). Considering the aforementioned constraints, universal yield–density models can be developed to predict AOPD, as long as the yield data distribution is known. The latter would allow constructing mathematical frameworks to upscale site-specific yield–density models to larger spatial domains (e.g., county, district, and regional scales). The challenge limiting the applicability of these models remains in the factors defining yield environments, therefore, future research should focus on improving understanding of the key drivers contributing to yield environments.

ACKNOWLEDGMENTS

This study was supported by CAPES Foundation, Ministry of Education of Brazil, Brasilia- DF, 70.040-020, Aquarius project (http://w3.ufsm.br/projetoaquarius/index.php/pt/), and Kansas Corn Commission. This is contribution no. 17-380-J from the Kansas Agricultural Experiment Station and process 88887.130848/2016-00 from CAPES.

REFERENCES

Al-Kaisi, M.M., and X. Yin. 2003. Effects of nitrogen rate, irrigation rate, and plant population on corn yield and water use efficiency. Agron. J. 95:1475–1482. doi:10.2134/agronj2003.1475

Arnall, D.B., A.P. Mallarino, M.D. Ruark, G.E. Varvel, J.B. Solie, M.L. Stone, J.L. Mullock, R.K. Taylor, and W.R. Raun. 2013. Relationship between grain crop yield potential and nitrogen response importance of grain yield potential for making nitrogen recommendations. Agron. J. 105:1335–1344. doi:10.2134/agronj2013.0034

Assefa, Y., P.V.V. Prasad, P. Carter, M. Hinds, G. Bhalla, R. Schon, M. Jeschke, S. Paszkiewicz, and I.A. Ciampitti. 2017. A new insight into corn yield: Trends from 1987 through 2015. Crop Sci. 57:2799–2811. doi:10.2135/cropsci2017.01.0066

Assefa, Y., P.V.V. Prasad, P. Carter, M. Hinds, and G. Bhalla. 2016. Yield responses to planting density for us modern corn hybrids: A synthesis-analysis. Crop Sci. 56:2802–2817. doi:10.2135/cropsci2016.04.0215

Barnhisel, R.I., M.J. Bitzer, J.H. Grove, and S.A. Shearer. 1996. Agronomic benefits of varying corn seed populations: A central Kentucky study. In: P.C. Roberts, R.H. Rust, and W.E. Larson, editors, Precision agriculture. American Society of Agronomy, Crop Science Society of America, Soil Science Society of America, Madison, WI. p. 957–965.

Bender, R.R., J.W. Haegele, M.L. Ruffo, and F.E. Below. 2013. Nutrient uptake, partitioning, and remobilization in modern, transgenic insect-protected maize hybrids. Agron. J. 105:161–170. doi:10.2134/agronj2012.0352

Blumenthal, J.M., D.J. Lyon, and W.W. Stroup. 2003. Optimal plant population and nitrogen fertility for dryland corn in western Nebraska. Agron. J. 95:878–883. doi:10.2134/agronj2003.0878

Bruns, H.A., and H.K. Abbas. 2005. Ultra-high plant populations and nitrogen fertility effects on corn in the Mississippi Valley. Agron. J. 97:1136–1140. doi:10.2134/agronj2004.0295

Bullock, D.G., D.S. Bullock, E.D. Nafziger, T.A. Doerge, S.R. Paszkiewicz, P.R. Carter, and T.A. Peterson. 1998. Does variable rate seeding of corn pay? Agron. J. 90:830–836. doi:10.2134/agronj1998.00021962009000060019x

Cerrato, M.E., and A.M. Blackmer. 1990. Comparison of models for describing corn yield response to nitrogen fertilizer. Agron. J. 82:138–143. doi:10.2134/agronj1990.00021962008200010030x

Ciampitti, I.A., J.J. Camberato, S.T. Murrell, and T.J. Vyn. 2013. Maize nutrient accumulation and partitioning in response to plant density and nitrogen rate: I. macronutrients. Agron. J. 105:783–795. doi:10.2134/agronj2012.0467

Ciampitti, I.A., and T.J. Vyn. 2011. A comprehensive study of plant density consequences on nitrogen uptake dynamics of maize plants from vegetative to reproductive stages. F. Crop. Res. 121:2–18. doi:10.1016/j.fcr.2010.10.009

Ciampitti, I.A., and T.J. Vyn. 2012. Physiological perspectives of changes over time in maize yield dependency on nitrogen uptake and associated nitrogen efficiencies: A review. F. Crop. Res. 133:48–67. doi:10.1016/j.fcr.2012.03.008

Ciampitti, I.A., and T.J. Vyn. 2014. Understanding global and historical nutrient use efficiencies for closing maize yield gaps. Agron. J. 106:2107–2117. doi:10.2134/agronj14.0025

Colville, W.L., A. Dreier, D.P. McGill, P. Grabouski, and P. Ehlers. 1964. Influence of plant population, hybrid, and “productivity level” on irrigated corn production. Agron. J. 56:332–335. doi:10.2134/agronj1964.00021962005600030024x

Córdoba, M.A., C.I. Bruno, J.L. Costa, N.R. Peralta, and M.G. Balzarini. 2016. Protocol for multivariate homogeneous zone delineation in precision agriculture. Biosystems Eng. 143:95–107. doi:10.1016/j.biosystemseng.2015.12.008

Crawley, M.J. 2014. Statistics: An introduction using R. John Wiley & Sons, Chichester, UK.

Dhital, S., and W.R. Raun. 2016. Variability in optimum nitrogen rates for maize. Agron. J. 108:2165–2173. doi:10.2134/agronj2016.03.0139

Duvick, D.N. 1997. What is yield? In: G.O. Edmeades, editor, Developing drought and low N-tolerant maize. CIMMYT, El Batan, Mexico. p. 332–335.

Duvick, D.N. 2005. The contribution of breeding to yield advances in maize (*Zea mays* L.). Adv. Agron. 86:83–145. doi:10.1016/S0065-2113(05)86002-X

Duvick, D.N., and K. Cassman. 1999. Post–green revolution trends in yield potential of temperate maize in the United States. Crop Sci. 39:1622–1630. doi:10.2135/cropsci1999.3961622x

Eck, H. 1984. Irrigated corn yield response to nitrogen and water. Agron. J. 76:421–428. doi:10.2134/agronj1984.00021962007600030014x

Edwards, J.T., L.C. Purcell, and E.D. Vories. 2005. Light interception and yield potential of short-season maize (*Zea mays* L.) hybrids in the Midsouth. Agron. J. 97:225–234.

Ferguson, R.B., G.W. Hergert, J.S. Schepers, C. Gotway, J.E. Cahoon, and T. Peterson. 2002. Site-specific nitrogen management of irrigated maize yield and soil residual nitrate effects. Soil Sci. Soc. Am. J. 66:544–553. doi:10.2136/sssaj2002.5440

Franzen, D., N. Kitchen, K. Holland, J. Schepers, and W. Raun. 2016. Algorithms for in-season nutrient management in cereals. Agron. J. 108:1775–1781. doi:10.2134/agronj2016.01.0041

Harri, A., C. Erdem, K.H. Coble, and T.O. Knight. 2009. Crop yield distributions: A reconciliation of previous research and statistical tests for normality. Rev. Agric. Econ. 31:163–182. doi:10.1111/j.1467-9353.2008.01431.x

Hennessy, D.A. 2009. Crop yield skewness and the normal distribution. J. Agric. Resour. Econ. 34:34–52.

Holland, K., and J. Schepers. 2010. Derivation of a variable rate nitrogen application model for in-season fertilization of corn. Agron. J. 102:1415–1424. doi:10.2134/agronj2010.0015

Hörbe, T.A.N., T.J.C. Amado, A.O. Ferreira, and P.J. Alba. 2013. Optimization of corn plant population according to management zones in Southern Brazil. Precis. Agric. 14:450–465. doi:10.1007/s11119-013-9308-7

Inman, D., R. Khosla, D.G. Westfall, and R. Reich. 2005. Nitrogen uptake across site specific management zones in irrigated corn production systems. Agron. J. 97:169–176. doi:10.2134/agronj2005.0169

Jaynes, D.B., T.C. Kaspar, and T.S. Colvin. 2011. Economically optimal nitrogen rates of corn: Management zones delineated from soil and terrain attributes. Agron. J. 103:1026–1035. doi:10.2134/agronj2010.0472

Koch, B., R. Khosla, W.M. Frasier, D.G. Westfall, and D. Inman. 2004. Economic feasibility of variable-rate nitrogen application utilizing site-specific management zones. Agron. J. 96:1572–1580. doi:10.2134/agronj2004.1572

Li, J., R.Z. Xie, K.R. Wang, B. Ming, Y.Q. Guo, G.Q. Zhang, and S.K. Li. 2015. Variations in maize dry matter, harvest index, and grain yield with plant density. Agron. J. 107:829–834. doi:10.2134/agronj14.0522

Licht, M.A., A.W. Lenssen, and R.W. Elmore. 2016. Corn (*Zea mays* L.) seeding rate optimization in Iowa, USA. Precis. Agric. 18:452–469. doi:10.1007/s11119-016-9464-7

Long, Q. 2015. varComp: Variance component models. https://cran.r-project.org/package=varComp (accessed 2 Oct. 2017).

Maddonni, G.A., and M.E. Otegui. 2004. Intra-specific competition in maize: Early establishment of hierarchies among plants affects final kernel set. Field Crops Res. 85:1–13. doi:10.1016/S0378-4290(03)00104-7

Moser, S.B., B. Feil, S. Jampatong, and P. Stamp. 2006. Effects of preanthesis drought, nitrogen fertilizer rate, and variety on grain yield, yield components, and harvest index of tropical maize. Agric. Water Manage. 81(1–2):41–58. doi:10.1016/j.agwat.2005.04.005

Otegui, M.E., and S. Melón. 1997. Kernel set and flower synchrony within the ear of maize: I. Sowing date effects. Crop Sci. 37:441–447. doi:10.2135/cropsci1997.0011183X003700020023x

Peralta, N., Y. Assefa, J. Du, C. Barden, and I. Ciampitti. 2016. Mid-season high-resolution satellite imagery for forecasting site-specific corn yield. Remote Sens. 8:848. doi:10.3390/rs8100848

Peralta, N.R., J.L. Costa, M. Balzarini, M. Castro Franco, M. Córdoba, and D. Bullock. 2015. Delineation of management zones to improve nitrogen management of wheat. Comput. Electron. Agric. 110:103–113. doi:10.1016/j.compag.2014.10.017

Pierce, F.J., and P. Nowak. 1999. Aspects of precision agriculture. Adv. Agron. 67:1–85.

Ping, J.L., R.B. Ferguson, and A. Dobermann. 2008. Site-specific nitrogen and plant density management in irrigated maize. Agron. J. 100:1193–1204. doi:10.2134/agronj2007.0174

Pinheiro, J., D. Bates, S. DebRoy, D. Sarkar, and R Core Team. 2017. nlme: Linear and nonlinear mixed effects models. R package version 3.1-131. https://CRAN.R-project.org/package=nlme (accessed 2 Oct. 2017).

R Core Team. 2017. R: A language and environment for statistical computing. https://www.r-project.org/ (accessed 2 Oct. 2017).

Raun, W.R., J.B. Solie, and M.L. Stone. 2011. Independence of yield potential and crop nitrogen response. Precis. Agric. 12:508–518. doi:10.1007/s11119-010-9196-z

Raun, W.R., J.B. Solie, M.L. Stone, K.L. Martin, K.W. Freeman, R.W. Mullen, H. Zhang, J.S. Schepers, and G.V. Johnson. 2005. Optical sensor-based algorithm for crop nitrogen fertilization. Commun. Soil Sci. Plant Anal. 36(19–20):2759–2781. doi:10.1080/00103620500303988

Roberts, D.F., R.B. Ferguson, N.R. Kitchen, V.I. Adamchuk, and J.F. Shanahan. 2012. Relationships between soil-based management zones and canopy sensing for corn nitrogen management. Agron. J. 104:119–129. doi:10.2134/agronj2011.0044

Rossini, M.A., G.A. Maddonni, and M.E. Otegui. 2012. Inter-plant variability in maize crops grown under contrasting N × stand density combinations: Links between development, growth and kernel set. Field Crops Res. 133:90–100. doi:10.1016/j.fcr.2012.03.010

Sangoi, L., M.A. Gracietti, C. Rampazzo, and P. Bianchetti. 2002. Response of Brazilian maize hybrids from different eras to changes in plant density. Field Crops Res. 79:39–51. doi:10.1016/S0378-4290(02)00124-7

Sarlangue, T.S., F.H. Andrade, P.A. Calviño, and L.C. Purcell. 2007. Why do maize hybrids respond differently to variations in plant density? Agron. J. 99:984–991. doi:10.2134/agronj2006.0205

Scharf, P.C., N.R. Kitchen, K.A. Sudduth, and J.G. Davis. 2006. Spatially variable corn yield is a weak predictor of optimal nitrogen rate. Soil Sci. Soc. Am. J. 70:2154–2160. doi:10.2136/sssaj2005.0244

Scharf, P.C., N.R. Kitchen, K.A. Sudduth, J.G. Davis, V.C. Hubbard, and J.A. Lory. 2005. Field-scale variability in optimal nitrogen fertilizer rate for corn. Precis. Agric. 97:452–461.

Schepers, A.R., J.F. Shanahan, M.a. Liebig, J.S. Schepers, S.H. Johnson, and A. Luchiari. 2004. Appropriateness of management zones for characterizing spatial variability of soil properties and irrigated corn yields across years. Agron. J. 96:195–203. doi:10.2134/agronj2004.0195

Schmidt, J.P., A.J. DeJoia, R.B. Ferguson, R.K. Taylor, R.K. Young, and J.L. Havlin. 2002. Corn yield response to nitrogen at multiple in-field locations. Agron. J. 94:798–806. doi:10.2134/agronj2002.7980

Setiyono, T.D., H. Yang, D.T. Walters, A. Dobermann, R.B. Ferguson, D.F. Roberts, D.J. Lyon, D.E. Clay, and K.G. Cassman. 2011. Maize-N: A decision tool for nitrogen management in maize. Agron. J. 103:1276–1283. doi:10.2134/agronj2011.0053

Shanahan, J.F., T.A. Doerge, J.J. Johnson, and M.F. Vigil. 2004. Feasibility of site-specific management of corn hybrids and plant densities in the great plains. Precis. Agric. 5:207–225. doi:10.1023/B:PRAG.0000032762.72510.10

Shanahan, J.F., N.R. Kitchen, W.R. Raun, and J.S. Schepers. 2008. Responsive in-season nitrogen management for cereals. Comput. Electron. Agric. 61:51–62. doi:10.1016/j.compag.2007.06.006

Shapiro, C.A., and C.S. Wortmann. 2006. Corn response to nitrogen rate, row spacing, and plant density in eastern Nebraska. Agron. J. 98:529–535. doi:10.2134/agronj2005.0137

Solie, J.B., A. Dean Monroe, W.R. Raun, and M.L. Stone. 2012. Generalized algorithm for variable-rate nitrogen application in cereal grains. Agron. J. 104:378–387. doi:10.2134/agronj2011.0249

Stanger, T.F., and J.G. Lauer. 2006. Optimum plant population of Bt and non-Bt corn in Wisconsin. Agron. J. 98:914–921. doi:10.2134/agronj2005.0144

Tokatlidis, I.S., V. Has, V. Melidis, I. Has, I. Mylonas, G. Evgenidis, A. Copandean, E. Ninou, and V.A. Fasoula. 2011. Maize hybrids less dependent on high plant densities improve resource-use efficiency in rainfed and irrigated conditions. Field Crops Res. 120:345–351. doi:10.1016/j.fcr.2010.11.006

Tokatlidis, I.S., and S.D. Koutroubas. 2004. A review of maize hybrids' dependence on high plant populations and its implications for crop yield stability. F. Crop. Res. 88(2–3):103–114. doi:10.1016/j.fcr.2003.11.013

Tollenaar, M., and E.A. Lee. 2002. Yield potential, yield stability and stress tolerance in maize. Field Crops Res. 75:161–169. doi:10.1016/S0378-4290(02)00024-2

Tollenaar, M., and J. Wu. 1999. Yield improvement in temperate maize is attributed to greater stress tolerance. Crop Sci. 39:1597–1604. doi:10.2135/cropsci1999.3961597x

van Averbeke, W., and J.N. Marais. 1994. Maize response to plant population and soil water supply: II. Plant barrenness and harvest index. South African J. Plant Soil 11:84–89. doi:10.1080/02571862.1994.10634300

Van Roekel, R.J., and J.A. Coulter. 2011. Agronomic responses of corn to planting date and plant density. Agron. J. 103:1414–1422. doi:10.2134/agronj2011.0071

Widdicombe, W., and K. Thelen. 2002. Row width and plant density effects on corn grain production in the Northern corn belt. Agron. J. 94:1020–1023. doi:10.2134/agronj2002.1020

Woli, K.P., C. Lee Burras, L.J. Abendroth, and R.W. Elmore. 2014. Optimizing corn seeding rates using a field's corn suitability rating. Agron. J. 106:1523–1532. doi:10.2134/agronj14.0054

Yan, P., J. Pan, W. Zhang, J. Shi, X. Chen, and Z. Cui. 2017. A high plant density reduces the ability of maize to use soil nitrogen. PLoS One 12:1–12. doi:10.1371/journal.pone.0172717